Linear term correction for needlets/wavelets non-Gaussianity estimator

Simona Donzelli, Frode Hansen, Domenico Marinucci
and C. Baccigalupi, M. Liguori, S. Matarrese, A. Renzi

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The variance of the wavelet cubic estimator

wavelet statistic: $\frac{1}{4\pi}\frac{1}{\sigma_i\sigma_j\sigma_k}\int_{S^2} w(R_i,n_1)w(R_j,n_1)w(R_k,n_1)dn_1$

$w(R_i,n_1)$ = wavelet coefficient scale R_i location n_1

variance if full-sky maps with isotropic noise:

$$\langle w(R_i,n_1)w(R_j,n_1)w(R_k,n_1)w(R_r,n_2)w(R_s,n_2)w(R_t,n_2)\rangle$$

$$= \langle w(R_i,n_1)w(R_r,n_2)\rangle \langle w(R_j,n_1)w(R_s,n_2)\rangle$$
$$\times \langle w(R_k,n_1)w(R_t,n_2)\rangle + 5 \text{ permutations}$$

pair with n_1, n_1 or n_2, n_2 vanish

The variance of the wavelet cubic estimator

wavelet statistic: $\frac{1}{4\pi}\frac{1}{\sigma_i\sigma_j\sigma_k}\int_{S^2} w(R_i,n_1)w(R_j,n_1)w(R_k,n_1)dn_1$

$w(R_i,n_1)$ = wavelet coefficient scale R_i location n_1

variance if <u>masked maps and anisotropic noise</u>:

$$\langle w(R_i,n_1)w(R_j,n_1)w(R_k,n_1)w(R_r,n_2)w(R_s,n_2)w(R_t,n_2)\rangle$$

$$= \langle w(R_i,n_1)w(R_r,n_2)\rangle \langle w(R_j,n_1)w(R_s,n_2)\rangle$$
$$\times \langle w(R_k,n_1)w(R_t,n_2)\rangle + 14 \text{ permutations}$$

pair with n_1, n_1 or n_2, n_2 do NOT vanish:
we need a linear correction!

The linear correction term

$$\int_{S^2} w(R_i,n)w(R_j,n)w(R_k,n)dn$$

$$\Downarrow$$

Wick products:

$$\int_{S^2} (: w(R_i,n)w(R_j,n)w(R_k,n) :)\, dn$$

i.e. adding the linear term :=

$$-\int_{S^2} \langle w(R_i,n)w(R_j,n)\rangle\, w(R_k,n)dn$$

$$-\int_{S^2} \langle w(R_i,n)w(R_k,n)\rangle\, w(R_j,n)dn$$

$$-\int_{S^2} \langle w(R_k,n)w(R_j,n)\rangle\, w(R_i,n)dn$$

it's analogous for needlet bispectrum

application: V+W cleaned maps WMAP7

WMAP optimal result: $f_{NL}^{local} = 32 \pm 21$ (Komatsu et al. 2011)

standard needlets*: * no optimal coadding

linear term	f_{NL}^{local}	sigma	f_{NL}^{NGsims}
NO	43.6	24.9	30.7
YES	39.3	22.1	31.2

mexican needlets*:

linear term	f_{NL}^{local}	sigma	f_{NL}^{NGsims}
NO	37.8	25.4	31.4
YES	26.6	22.2	31.9

sigma = standard deviation over 10200 simulations

f_{NL}^{NGsims} = f_{NL} over 300 NG simulations with f_{NL}=30

application: V+W cleaned maps WMAP7

SMHW*:

* no optimal coadding

linear term	f_{NL}^{local}	sigma	f_{NL}^{NGsims}
NO	77.8	27.3	31.8
YES	33.1	21.9	29.5

SMHW scale-by-scale mean subtracted*:

linear term	f_{NL}^{local}	sigma	f_{NL}^{NGsims}
NO	37.5	22.3	30.1
YES	34.4	22.0	30.1

the linear term is approximated by mean subtraction

it can be analytically shown that this is true if noise is not very anisotropic

conclusions

- the needlet bispectrum with the linear term is <u>very close to an optimal f_{NL} estimator</u>

- where noise anisotropy is not very relevant the linear term can be approximated by mean subtraction

- for details: paper near to submission

- next step: it's time to move back to Planck!