Analysis of high-dispersion tracer tests in cores containing polymer gels

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Abstract

The analysis of a tracer experiment requires care when one of the fluid phases within the porous medium is immobile but miscible with the tracer carrier fluid. This situation gives rise to high levels of dispersion ($Pe < 10$) and interphase mass transfer. We show that applying the classical analytical expression for the tracer concentration in an infinite one-dimensional core introduces significant mass balance errors at low Peclet numbers, even though a good fit to the experimental data may be obtained. Next, we present a rapid test of whether mass transfer into the immobile phase is important. The test is based on the observation that theoretical effluent concentration histories from a finite core in the absence of interphase mass transfer span a well-defined and relatively limited area on a plot of concentration vs. time. If an experimental effluent history does not lie within this area, a mass transfer model must be applied. Finally, we show that at high mass transfer rates, the effluent history becomes insensitive to the volume fractions of the miscible fluid phases. Thus, it can be difficult to determine a unique set of fitting parameters for experiments in this regime. To avoid this difficulty, experiments designed to determine miscible, immobile phase volumes should be conducted at high flow rates (low residence times) or with slowly diffusing tracers. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Tracer tests are widely used in the laboratory and in the field to characterise the degree of dispersion associated with flow through a porous medium. Another important application is the determination of volume fractions of fluids occupying the pore space. This application depends upon tracer molecules transferring from the carrier phase into the fluid phase of interest. Examples include evaluating residual oil saturations (Allison et al., 1991), determining volumes of organic contaminants in soil or aquifers (Jin et al., 1995), estimating excluded pore volume for flow of polymer solutions (Zaitoun and Kohler, 1988; Sorbie and Huang, 1991) and quantifying the...
volume of ‘dead-end’ pores (Coats and Smith, 1964). Another example arises in studies of water shutoff treatments using polymer gels, where it is often of interest to determine the volume fraction of the pore space occupied by polymer gel, or the volume fraction open to flow (Liang et al., 1992).

When applied and analysed properly, tracer tests are simple, reliable and inexpensive methods for probing porous media. Indeed, for some field applications, they represent the only method of assessing large soil or rock volumes. This paper focuses on the laboratory determination of the volume fractions of pore space occupied by various fluid phases, with particular reference to the case when one phase is a water-based polymer gel. Mathematical development is relegated to Appendices A and B.

2. Results

In this section, we present examples of laboratory data that demonstrate the application of the mathematical models described in Appendices A and B. The examples involve aqueous tracers in cores containing polymer gels and illustrate the care which must be taken in this situation. All examples involve a single step change in injected tracer concentration.

2.1. Example 1: core containing gel and residual oil

Fig. 1 shows the effluent history of a tracer injected into a core after establishing residual oil saturation, injecting gelant and allowing it to gel. The time axis of Fig. 1 is scaled by the total pore volume of the core. The aqueous tracer used did not

Fig. 1. Effluent history for a core containing brine, gel and residual oil (Example 1). Experimental data are compared with curves calculated from infinite-core, no-mass-transfer model (Eq. 9) and from finite-core, mass-transfer model (numerical solution to Eq. (1)). While the infinite-core solution provides a good fit, it cannot be accepted. This is because it neglects the high fluxes due to dispersion and therefore violates the overall mass balance significantly (see text).
partition into the residual oil phase. The purpose of this test was to determine the volume open to brine flow when brine was forced through the core.

Let us compare how well the various models fit the data. The simplest assumptions are that (i) infinite-core boundary conditions are applicable and that (ii) mass transfer to the gel phase was negligible. The corresponding solution, Eq. (9), fits the data for $Pe = 0.56$ and $f = 0.368$. Although the fit is reasonably good, particularly at early times, it cannot be accepted, because the infinite-core solution does not conserve mass when applied to experiments with such high levels of dispersion. This can be shown by calculating the overall mass balance, Eq. (15), for the effluent history computed from Eq. (9). In this case, the left hand side of Eq. (15) reduces to $f$, rather than $f + S_g$, because mass transfer into the gel phase was assumed to be negligible in order to apply Eq. (9). The mass balance for this infinite-core solution yielded $f = 0.608$. But the solution itself was computed with $f = 0.368$, which indicates an error in the mass balance by $0.608 - 0.368 = 65\%$. This error arises because the contribution of dispersive flux at the core inlet is neglected in the infinite-core approximation (cf. Eqs. (3) and (4)).

This shows that infinite-core boundary conditions cannot be applied for this tracer test. Let us now apply the finite-core boundary conditions but continue to assume that mass transfer is negligible; in this case the effluent history is given by Eq. (10). As described in Appendices A and B, Eq. (10) also defines a region in the concentration–time plane in which all effluent histories for these particular boundary conditions must fall. As shown in Fig. 2, the effluent concentration data for this tracer test fall outside this region of physical validity. It follows

![Fig. 2. The experimental data of Example 1 (core containing brine, gel and residual oil) lie outside the envelope of curves calculated from finite-core, no-mass-transfer model (Eq. (10), with $f = 0.31$), indicating that mass transfer into the gel must have occurred. While the plot is for a particular value of $f$, in fact there is no value of $f$ such that the data lie within the envelope.](image-url)
that this tracer test cannot be modeled with finite-core boundary conditions and no mass transfer. Plots such as Fig. 2 are convenient yet powerful graphical means for evaluating tracer behaviour.

This analysis of no-mass-transfer models, with finite- and infinite-core boundary conditions, shows that tracer must have transferred into the gel during the experiment. Although this is not surprising, the point is worth emphasising. Unwarranted application of the infinite-core no-mass-transfer solution is tempting because of its familiarity and the relative ease with which it can be made to fit to a wide range of data. Because its misuse can lead to substantial errors, deeper investigation is always recommended when using simplifying assumptions.

We conclude that in order to interpret the data of this example, the mass-transfer model for a finite core must be used. A best fit (which also proved to be unique) was found for the following values of the parameters: $f = 0.302$, $Pe = 0.96$ and $St = 1.72$. The corresponding effluent history is shown in Fig. 1. Because we assume that tracer molecules do not diffuse into the oil phase, it is possible to calculate the residual oil saturation directly from the experimental data; in the example reported in Fig. 1, the calculated residual saturation is $S_o = 0.313$; therefore the volume fraction occupied by the gel is $S_g = 1 - S_o - f = 0.385$.

2.2. Example 2: core containing a syneresis-prone gel

In this example, the purpose of the tracer test was to assess the volume open to brine flow after a syneresis-prone gel had been aged in the core. The free volume thus obtained would determine the ex-

![Graph](image)

Fig. 3. Example 2: effluent history for a brine-saturated core containing syneresis-prone gel. Experimental data are compared with calculated curves obtained by fitting the data with different models.
tent to which the gel had syneresed within the pore space (Bryant et al., 1996). Fig. 3 shows the observed effluent history. The infinite-core, no-mass-transfer model, Eq. (9), fits the data when $f$ is taken to be 0.8 and $Pe = 2.98$. As pointed out above, a solution based on infinite-core boundary conditions should not be accepted without checking the mass balance, and in this case a check using Eq. (15) reveals an error of 57%.

With the finite-core no-mass-transfer solution, Eq. (10), we obtained a good fit for $f = 1$ and $Pe = 2.408$ (see Fig. 3). This contrasts with the previous example, in which the no-mass-transfer model was physically inconsistent with the data. On the other hand, a value of $f = 1$ would imply that the gel has syneresed to zero volume, which is objectionable on physical grounds. It is thus of interest to suppose that the syneresed gel occupies a non-negligible fraction of the pore space and to apply the finite-core mass-transfer model, Eq. (1), with $S_{gel} = 1 - f$. A better fit, obtained by trial and error, was found with $Pe = 2$, $St = 5$ and $f = 0.6$ (see Fig. 3).

The better fit with the mass transfer model is not surprising, if only because it offers an extra fitting parameter. It is unsettling, however, that fits of comparable quality can be obtained with quite different values for $f$ ($f = 1$ in the no-mass-transfer model and $f = 0.6$ in the mass transfer model). This observation raises concerns about uniqueness, and this concern was reinforced when an automated fitting routine was applied to this tracer test. The routine showed poor convergence behaviour, and the set of parameters ($Pe$, $St$, $f$) that the routine returned was strongly dependent upon the initial guess for the parameters. This led to a more extensive investigation of the problem of uniqueness, which is discussed in Section 3. The conclusion with respect to this tracer test is that a definitive value of $f$, and therefore of the extent of syneresis of the gel within the core, cannot be determined.

### 3. The problem of uniqueness

Fitting a multi-parameter model to data always raises the question of whether the best fit is unique. An example of this difficulty appears in Fig. 4, where the effluent histories for the two phase convection–dispersion–mass transfer model (Eq. (1) with $S_{gel} = 1 - f$) are plotted for four sets of parameters. Each history has $Pe = 2$, a value of $f$ between 0.5 and 0.8, and a value of $St$ between 3 and 7. The curves are scarcely distinguishable from the curve which fit the data of Example 2 (cf. Fig. 3), and therefore any value between 0.5 and 0.8 could be plausible for the volume fraction open to convection.

The mass transfer model, Eq. (1), assumes that tracer molecules move into the gel volume at a rate proportional to bulk concentration differences. Although this ad hoc model is reasonable, it has not been verified independently. Thus, there are no compelling physical grounds to prefer any of the curves of Fig. 4, even though a single curve could be identified as “best” in the sense of exhibiting the smallest difference from the data.

In this case, the non-uniqueness has a physical basis. The relatively high rate of mass transfer ($St > 1$) means that the tracer is sampling the gel volume roughly as fast as it samples the free volume, so that the tracer sees the entire pore space regardless of how it is subdivided into gel and free volume. The high dispersion ($Pe = 2$) tends to smear out whatever signatures might otherwise have arisen from the differences in $St$ and $f$. A survey of effluent histories for a wide range of ($Pe$, $St$, $f$) values confirms this physical reasoning. When the rate of mass transfer is low, $St < 0.1$, the volume fraction of the immobile phase produces a strong signature, and effluent histories at different free volume fractions (e.g., differing by 0.1) are distinguishable. Thus, at low Stanton numbers, one can identify a meaningful best fit, or at worst a narrow range of parameter values which describe the data. At all rates of mass transfer, decreasing the degree of dispersion (increasing $Pe$) enhances the difference between effluent histories. However, this effect is weaker than the effect of reducing the rate of transfer. At high values of $f$ (> 0.8) the onset of non-uniqueness occurs when $St$ increases to values near unity; at lower values of $f$ (ca. 0.5) uniqueness may persist for $St$ as large as 10.

In practical terms, this means that a tracer test must be carried out at relatively low mass transfer rates in order to distinguish an immobile miscible phase. This suggests using tracers with low diffusion coefficients (e.g., large molecules) and running tests...
Fig. 4. At high rates of mass transfer, different sets of parameters can yield practically indistinguishable effluent histories from Eq. (1), so that a unique fit to data from Example 2 cannot be obtained.

at high flow rates (low residence times). In practice, the conveniently available range of diffusion coefficients may not be wide enough to effect the large changes in $St$ (an order of magnitude or more) that may be necessary. Furthermore, in the case of gels, the high resistance to flow can make high flow rates difficult to achieve.

4. Conclusions

Tracer experiments can be useful for determining pore volume fractions occupied by fluid phases. The interpretation of laboratory tracer tests in cores containing aqueous polymer gels raises two issues that are less commonly encountered in other tracer applications. First, high levels of dispersion ($Pe < 10$) can arise. Second, aqueous tracers will diffuse into the immobile gel phase.

At high levels of dispersion, effluent histories computed from the classical error function solution for the tracer concentration in an infinite core exhibit significant errors in the tracer mass balance, regardless of whether mass transfer is considered. Because the infinite-core solution is sufficiently flexible to fit a wide variety of effluent concentration histories, an explicit check of mass conservation is recommended, since the inappropriateness of the solution may not be evident in the apparent quality of the fit to the data. These considerations apply to any tracer test displaying high dispersion, not only to tests in cores containing gels.

As the rate of mass transfer during a tracer test is not known a priori it is useful to have a means of checking whether mass transfer can be neglected in the interpretation of the experiment. The analytical expression for effluent tracer concentration from a finite core in the absence of mass transfer provides
well-defined limits (at infinite dispersion, $Pe = 0$ and zero dispersion, $Pe = +\infty$) on physically valid effluent concentration histories. Comparing data with these limits provides a simple check of whether mass transfer was significant during the experiment.

When mass transfer into the immobile phase is sufficiently fast ($St$ of order unity or more, depending on $f$), the effluent concentration history becomes insensitive to the phase volume fractions. If the immobile phase is miscible with the carrier fluid, a non-partitioning tracer can usually be injected to resolve the volume fractions. When the immobile phase is immiscible with the tracer carrier fluid, as is the case with polymer gels and aqueous tracers, a non-partitioning tracer does not exist. Hence, an experiment with low Stanton number must be carried out, for example by using tracers with smaller diffusion coefficients or by conducting the test at higher flow rates. These conditions may be difficult to achieve in practice, however. The insensitivity to volume fractions at high $St$ raises the problem of non-uniqueness for parameter fitting routines. When an automated history matching package returns a relatively high value of $St$, the results should be checked for sensitivity to the initial guess, so that appropriate confidence limits (error bars) can be assigned to the parameters.

5. Nomenclature

$a = \text{specific interfacial area between gel phase and brine, } L^{-1}$
$A = \text{core cross-section, } L^2$
$c = C/C_0 \text{ normalised tracer concentration, dimensionless}$
$D = \text{effective diffusion constant (diffusion + dispersion) of tracer, } L^2 \, t^{-1}$
$f = \text{fraction of pore volume open to convection, dimensionless}$
$k = \text{mass transfer constant of proportionality, } L \, t^{-1}$
$L = \text{core length, } L$
$m = \text{mass of tracer, } m$
$P = Pe/4, \text{ dimensionless}$
$Pe = L^2/(D\tau), \text{ Peclet number, dimensionless}$
$Q = \text{flow rate, } L^3 \, t^{-1}$
$S = \text{fluid saturation, dimensionless}$

$St = k\tau, \text{ Stanton number, dimensionless}$
$t = \text{time, } t$
$t_0 = t/\tau, \text{ normalised time, dimensionless}$
$x = X/L, \text{ normalised distance from core inlet, dimensionless}$
$X = \text{distance from core inlet, } L$
$\phi = \text{porosity, dimensionless}$
$\lambda = \text{parameter in Eqs. (7) and (10), dimensionless}$
$\tau = AL\phi/Q, \text{ residence time, } t$

Subscripts

$b = \text{pertaining to brine phase}$
$gel = \text{pertaining to gel phase}$
$D = \text{dimensionless}$
$e = \text{residual oil}$
$efl = \text{effluent}$
$inj = \text{injected}$
$el = \text{eluted}$
$o = \text{pertaining to injected phase}$
$k = \text{index in infinite summations (Eq. (7))}$

Superscripts

fin = final

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Appendix A. Mathematical development

A.1. Physical system

Consider a one-dimensional porous medium containing up to three phases: brine, residual oil and an immobile water-based gel, as sketched in Fig. 5.

At time $t = 0$, brine containing a chemical tracer at concentration $C_0$ is injected into the core at a constant flow rate.

The following assumptions are made:

• Tracer molecules can diffuse from the brine into the gel phase but not into the oil phase.
Fig. 5. Tracer experiment in one-dimensional core. Brine carries a tracer through the core. The core also contains an immobile gel phase, into which the tracer can transfer, and a residual oil phase, into which the tracer does not transfer.

- The driving force for diffusion of the tracer into the gel is proportional to the difference in the concentrations in each phase; at equilibrium (no net mass transfer) the concentrations are assumed to be equal.
- Tracer within the gel phase does not diffuse along the core (in the direction of flow).

With these assumptions, the mass balances for tracer in the brine and gel phases take the following dimensionless forms:

$$\frac{\partial c_b}{\partial t_D} + \frac{1}{f} \frac{\partial c_b}{\partial x} - \frac{1}{Pe} \frac{\partial^2 c_b}{\partial x^2} + St(c_b - c_g) = 0$$

$$\frac{\partial c_g}{\partial t_D} = St \frac{f}{S_{gel}} (c_b - c_g).$$  \hspace{1cm} (1)

The saturations of gel and oil are denoted by $S_{gel}$ and $S_{or}$, respectively; the fraction of pore volume open to convection flow is denoted $f$ and we have $f + S_{gel} + S_{or} = 1$. Other terms are defined in Section 5.

Eq. (1) contains four parameters: $Pe$, $St$, $f$ and $S_{gel}$. In special or limiting cases, the number of parameters can be reduced, as shown in Table 1.

### A.2. Initial and boundary conditions

As the core is assumed to contain no tracer initially, we have the following initial condition:

$$c_b(x,0) = c_g(x,0) = 0.$$  \hspace{1cm} (2)

The injected concentration $C_i$ is assumed constant, corresponding to a step change tracer experiment; tracer pulses are not considered here. This implies that the tracer concentration inside the core approaches $C_i$ as $t \to \infty$, which in turn implies that the effluent concentration will also approach $C_i$.

Two sets of boundary conditions are considered, infinite core (Mannhardt and Nasr-El-Din, 1994):

$$c_b(x,t_D), c_g(x,t_D) \to 1 \quad x \to -\infty$$

$$c_b(x,t_D), c_g(x,t_D) \to 0 \quad x \to +\infty \hspace{1cm} (3)$$

and finite core (Brenner, 1962):

$$c_b(0^+, t_D) - \frac{1}{Pe} \left. \frac{\partial c_b}{\partial x} \right|_{x=0^+} = 1$$

$$\left. \frac{\partial c_b}{\partial x} \right|_{x=1} = 0. \hspace{1cm} (4)$$
Table 1
List of special and limiting cases to the more general system described by Eq. (1)

<table>
<thead>
<tr>
<th>Physical situation</th>
<th>Parameter values</th>
<th>Notes</th>
<th>Number of parameters in Eq. (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brine only</td>
<td>( St = 0; S_{sw} = S_{gel} = 0; f = 1 )</td>
<td>Reduces to classical convection–diffusion equation</td>
<td>1 (Pe)</td>
</tr>
<tr>
<td>Brine, residual oil</td>
<td>( St = 0; S_{gel} = 0; f = 1 - S_{sw} )</td>
<td>Can be reduced to classical convection–diffusion equation by rescaling ( t_D ) and ( Pe )</td>
<td>2 (Pe, f)</td>
</tr>
<tr>
<td>Brine, residual oil, gel, but no mass transfer into gel</td>
<td>( St = 0; S_{gel} = (1 - S_{ms}) - f )</td>
<td>Behaves as brine and residual oil</td>
<td>2 (Pe, f)</td>
</tr>
<tr>
<td>Brine, gel</td>
<td>( St &gt; 0; S_{sw} = 0; S_{gel} = 1 - f )</td>
<td>Eq. (1) reduces to three parameters: ( Pe, St ) and ( f )</td>
<td>3 (f, Pe, St)</td>
</tr>
<tr>
<td>Brine, gel, oil at known ( S_{ms} )</td>
<td>( St &gt; 0; S_{gel} = (1 - S_{ms}) - f )</td>
<td>Eq. (1) reduces to three parameters: ( Pe, St ) and ( f ).</td>
<td>3 (f, Pe, St)</td>
</tr>
</tbody>
</table>

A.3. Computation of the effluent history

The solution of Eq. (1) with appropriate boundary conditions gives the tracer concentration in the brine and gel phases as a function of time and position in the core. The concentration of tracer in the brine exiting the core is \( c_s(t_D) = c_s(x = 1, t_D) \), and this computed effluent concentration history can be compared to the experimental data.

In the case when \( f = 1 \) and \( St = 0 \) (brine only), Eq. (1) can be solved analytically for several boundary conditions (Mannhardt and Nasr-El-Din, 1994); the analytical solutions for the effluent histories derived with the conditions 3 and 4 are presented below. The solution can also be readily transformed to apply when \( f < 1 \) in the absence of mass transfer, as shown below.

When no analytical solution exists, we have resorted to numerical integration, usually via an implicit finite difference scheme. A procedure was written to assist in finding the optimal fit of effluent history data. The procedure used a \( \chi^2 \) minimisation routine, available in the programming language Mathematica, based on a steepest descent method.

A.4. The convection–dispersion equation: analytical solutions for different boundary conditions

The standard convection–dispersion equation can be readily obtained from Eq. (1) by imposing \( f = 1 \) and \( St = 0 \). It reads:

\[
\frac{\partial c}{\partial t_D} + \frac{\partial c}{\partial x} - \frac{1}{Pe} \frac{\partial^2 c}{\partial x^2} = 0
\]

In this section, we present some of the solutions for the effluent history, \( c_s(t_D) \), that can be derived analytically from Eq. (5).

A.4.1. Infinite core boundary conditions (Eq. (3))

When infinite core boundary conditions are applied in Eq. (5), the following solution for the effluent concentration is obtained (Mannhardt and Nasr-El-Din, 1994):

\[
c_s(t_D) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{\sqrt{Pe} \left( 1 - t_D \right)}{2 \sqrt{t_D}} \right) \right]
\]

A.4.2. Finite core boundary conditions (Eq. (4))

When finite core boundary conditions are applied the effluent concentration takes the following analytical form (Brenner, 1962)

\[
c_s(t_D) = \exp \left[ P (2 - t_D) \right] \times \sum_{k=1}^{\infty} \frac{\lambda_k \sin(2\lambda_k)}{\lambda_k^2 + P^2 + P} \exp \left( -\lambda_k^2 t_D/P \right)
\]

where

\[
P = \frac{Pe}{4},
\]

\[
\lambda_k = \begin{cases} \text{solution of } \lambda \tan \lambda = P & \text{for odd } k \\ \text{solution of } \lambda \cot \lambda = -P & \text{for even } k \end{cases}
\]

The use of Eq. (7) for the calculation of effluent histories requires some careful numerical handling, in particular:

- The non-linear equations that determine the terms \( \lambda_k \) must be solved numerically.
A.4.3. Extension to multiple phases in core

When other immobile phases are present in the core, but mass transfer of tracer from the carrier fluid into the other phases is still negligible, the mass balance takes the form

$$\frac{\partial c}{\partial t_d} + \int_{f}^{1} \frac{1}{\partial x} \frac{\partial c}{\partial x} - \frac{1}{P\epsilon} \frac{\partial^2 c}{\partial x^2} = 0. \quad (8)$$

After introducing the transformation \( t_0 = t_d/f \), \( P\epsilon' = P\epsilon/f \), Eq. (8) assumes the same form of Eq. (5) in the transformed time and Peclet number. Hence, the effluent history for infinite core boundary conditions is

$$c_e(t_d) = \begin{cases} 1 - \text{erf} \left( \frac{\sqrt{P\epsilon'} \left( 1 - t_D \right)}{2 \sqrt{t_D}} \right) & \text{for } t_0 \leq 1 \\ 1 - \text{erf} \left( \frac{\sqrt{P\epsilon/f} \left( 1 - (t_d/f) \right)}{2 \sqrt{t_d/f}} \right) & \text{for } t_0 > 1 \end{cases}$$

and for finite cores is

$$c_e(t_d) = \exp \left[ \frac{f}{\sqrt{\pi}} (2 - \frac{t_d}{f}) \right] \times \sum_{k=1}^{\infty} \frac{\lambda_k \sin(2\lambda_k)}{\lambda_k^2 + \left( \frac{f}{\sqrt{\pi}} \right)^2 + \frac{f}{\sqrt{\pi}}} \exp \left( -\lambda_k^2 t_d / P \right). \quad (9)$$

Appendix B. Derivation of the overall mass balance

Besides the differential mass balances derived above, an integral or overall mass balance is also convenient. It can be derived with the following steps:

- total mass in the gel within the core, \( m_g \): \( \int S_{\phi} A f_{c}^{u} C_g(X,t^{\text{fin}}) dX \);
- cumulative mass injected at time \( t^{\text{fin}} \), \( m_{inj} \): \( Q C_{0} t^{\text{fin}} \);
- cumulative mass eluted at time \( t^{\text{fin}} \), \( m_{el} \): \( Q \int_{0}^{t^{\text{fin}}} C_e(t) dt \).

Because at time \( t^{\text{fin}} \), the concentration is equal to \( C_0 \) everywhere, we have

$$\int_{0}^{t^{\text{fin}}} C_g(X,t^{\text{fin}}) dX = \int_{0}^{L} C_g(X,X^{\text{fin}}) dX = C_0 L \quad (11)$$

and, therefore, the overall mass balance \( m_b + m_g = m_{inj} - m_{el} \) (12) reduces to

$$fA \phi C_0 L + S_{\phi} A \phi C_0 L = QC_0 t^{\text{fin}} - Q \int_{0}^{t^{\text{fin}}} C_e(t) dt. \quad (13)$$

Eq. (13) can be rearranged to yield:

$$f + S_{\phi} = \frac{Q t^{\text{fin}}}{A \phi L} - \frac{Q}{A \phi L} \int_{0}^{t^{\text{fin}}} \frac{C_e(t)}{C_0} dt$$

$$= t_d^{\text{fin}} - \int_{0}^{t_d^{\text{fin}}} c_e(t_d) dt_d. \quad (14)$$

B.1. Mathematical consequences of finite-core equations

The analytical solution for the special case of a tracer test in a finite core in the absence of mass transfer (\( S_t = 0 \), see Eqs. (7) and (10)) provides a useful point of reference. Because the dispersion coefficient is a non-negative quantity, the Peclet number can only assume values in the range \([0, +\infty]\). Consequently, the portion of the \( \{c_e, t_0\} \) plane enclosed by the curves for \( Pe = 0 \) and \( Pe = +\infty \) defines the region where experimental effluent history curves must lie when \( S_t = 0 \), as sketched in Fig. 6 for the case when \( f = 1 \).

It is often useful to check measured effluent data against this region of physical validity; in this way experimental artefacts (e.g., errors in dead volume measurement) can be identified. This check can also be used to determine whether improper simplifications have been made (e.g., neglecting mass transfer) when multiple phases are present in the core. In the
latter case, $f$ will be less than unity and Eq. (10) must be used to determine the region of physical validity. If the data fall outside the limits defined by $0 \leq Pe < +\infty$, then mass transfer cannot be neglected in the data analysis.

B.2. Mathematical consequences of overall mass balance

Another useful formula arises from the long-term behaviour of the step change experiment, for which the dimensionless tracer concentrations $c_b$, $c_g$ and $c_e$ approach unity at large times. In this limit, the overall mass balance Eq. (14) reduces to a simple form:

$$f + S_{gel} = t_D^{fin} - \int_{0}^{t_D^{fin}} c_e(t_D) dt_D$$

where $t_D^{fin}$ is the dimensionless time when $c_e$ reaches unity.

When residual oil is present, Eq. (15) provides a simple method of estimating its saturation, $S_{w} = 1 - (f + S_{gel})$. This also fixes $S_{gel}$ in terms of $f$, thereby reducing the number of parameters in Eq. (1). Clearly, however, this method alone cannot be applied to determine the volume fraction occupied by the gel.

Eq. (15) also offers a simple check on the mass balance for computed effluent histories. This is particularly convenient for assessing whether infinite-core boundary conditions are applicable to an experiment, as described in the text for Example 1.

References