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Alexander Friedmann and the origins of modern cosmology

Friedmann, who died young in 1925, deserves to be called the father of Big Bang cosmology. But his seminal contributions have been widely misrepresented and undervalued.

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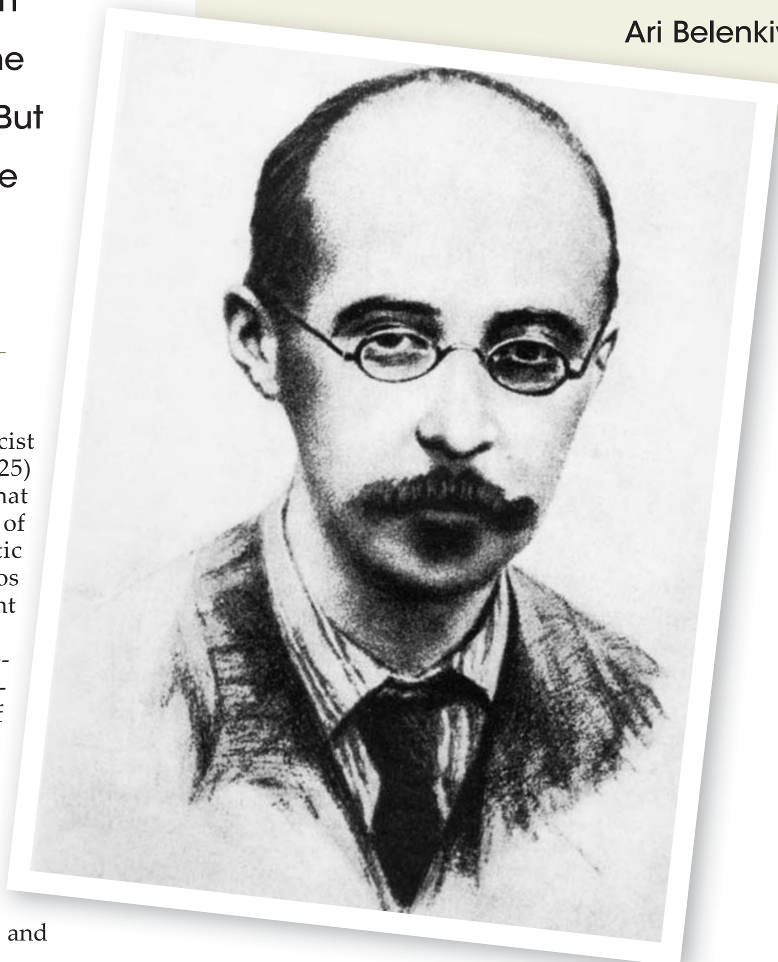

Figure 1. Alexander Friedmann in Petrograd, Soviet Union, in the early 1920s.

Ninety years ago, Russian physicist Alexander Friedmann (1888–1925) demonstrated for the first time that Albert Einstein's general theory of relativity (GR) admits nonstatic solutions. It can, he found, describe a cosmos that expands, contracts, collapses, and might even have been born in a singularity.

Friedmann's fundamental equations describing those possible scenarios of cosmic evolution provide the basis for our current view of the Big Bang and the accelerating universe. But his achievement initially met with strong resistance, and it has since then been widely misrepresented. In this article, I hope to clarify some persistent confusions regarding Friedmann's cosmological theory in the context of related work by contemporaries such as Einstein, Willem de Sitter, Arthur Eddington, and Georges Lemaître.

Last year's Nobel Prize in Physics was shared by three cosmological observers who discovered that the cosmic expansion is currently accelerating (see PHYSICS TODAY, December 2011, page 14). Thus, one of the scenarios introduced by Friedmann in 1922 and 1924 appears to correspond to reality.[1,2] The Royal Swedish Academy of Sciences background essay for the 2011 prize cites Friedmann's papers.[3] Regrettably, however, it significantly distorts his contributions.

Already in 1922, Friedmann had set the appropriate framework for a GR cosmology by introducing its most general metric and the "Friedmann equations," which describe the evolution of a perfect-fluid cosmos of uniform mass density ρ. And he elucidated all three major scenarios for a nonstatic universe consistent with GR. In fact, he introduced the expression "expanding universe" and estimated the period of an alternative periodic universe that's

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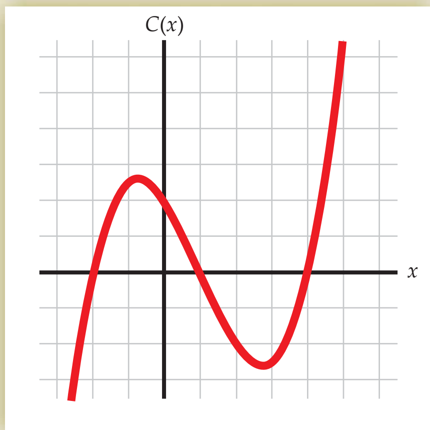
Figure 2. Friedmann's fundamental cubic $C(x)$, given by equation 4 in the text, is plotted here against cosmic radius x for arbitrary values of the free cosmological parameters λ and A. Their true values determine whether $C(x)$ has 0 or 2 positive roots, and they define three general scenarios of cosmic evolution. One gets additional limiting-case scenarios if the two positive roots are degenerate. Changing the sign of the linear term in $C(x)$ to describe a cosmos with negative curvature yields still more scenarios.



surprisingly close to what is believed now to be the time elapsed since the Big Bang. In 1924 he further revolutionized the discourse by presenting the idea of an infinite universe, static or nonstatic, with a constant negative curvature.

A short life

Born and raised in Saint Petersburg, Friedmann studied mathematics at the city's university.[4] There he attended the physics seminars of Vienna-born Paul Ehrenfest, who had moved to St. Petersburg with his Russian wife in 1907. After Friedmann graduated in 1910, he worked primarily in mathematical physics applied to meteorology and aerodynamics.

Following the outbreak of World War I in August 1914, Friedmann served with the Russian air force on the Austrian front as a ballistics instructor. He took part in several air-reconnaissance flights and was awarded the military cross. After the February 1917 revolution that deposed the czar, dozens of new universities were established across Russia and Friedmann obtained his first professorship, in Perm near the Ural Mountains.

At the end of the civil war that secured the Bolshevik regime in 1920, Friedmann (pictured in figure 1) returned to his hometown, renamed Petrograd, and started working as a physicist at the Geophysical Observatory. He soon became the observatory's director. Most of his personal research was oriented toward theories of turbulence and aerodynamics. But in parallel, he also worked on GR and quantum theory. A month before his untimely death from typhus in September 1925, Friedmann made a risky record-breaking balloon flight to collect high-altitude data.[4]

Einstein's 1905 special theory of relativity was well known in Russia. But awareness of Einstein's 1915 paper introducing GR was delayed because of the world war.[5] But soon after the war, news of the theory and of Eddington's confirmatory 1919 solar-eclipse observations caused tremendous excitement among scientists and the general public throughout Russia. And in 1921, the resumed shipment of European scientific publications provided scientists in Petrograd with access to the literature. Furthermore, physicist Vsevolod Frederiks brought insider information. Interned in Germany during the war, he worked at Göttingen as an assistant to David Hilbert, who independently proposed the GR equations early in 1916, not long after Einstein.

In collaboration with Frederiks, Friedmann wrote a mathematical introduction to GR. The first volume, devoted to tensor calculus, appeared in 1924. Another book, *The World as Space and Time*, written by Friedmann alone the year before, developed his philosophical interpretation of GR. But his fame rests on two *Zeitschrift für Physik* papers.[1,2] In them he introduced the fundamental idea of modern cosmology—that the geometry of the cosmos might be evolving, perhaps even from a singularity.

General relativity before Friedmann

The fundamental Einstein field equations of GR are

$$R_{\mu\nu} - g_{\mu\nu}R/2 - \lambda g_{\mu\nu} = -kT_{\mu\nu}\,, \qquad (1)$$

where the spacetime indices μ,ν run from 1 to 4 and the constant $k = 8\pi G/c^2$. The spacetime distribution of the energy–momentum tensor $T_{\mu\nu}$ determines the local geometry encoded in the metric tensor $g_{\mu\nu}$, and the "Ricci tensor" $R_{\mu\nu}$ is determined by $g_{\mu\nu}$ and its spacetime derivatives. The Ricci scalar R, a contraction of the Ricci tensor, is the actual local curvature of spacetime. The cosmological constant λ was introduced in 1917 by Einstein in the hope of finding a stable, static cosmological solution of the field equations.

To seek a cosmological solution for a homogeneous universe approximated by a perfect fluid of uniform density ρ and pressure P, one takes $T_{11} = T_{22} = T_{33} = -P$ and $T_{44} = c^2\rho$. All off-diagonal elements are zero. For simplicity, Einstein considered the cosmological approximation $P = 0$.

Finding cosmological solutions of the field equations required great ingenuity. Before 1922 only two simple solutions were discovered, one by Einstein and the other by de Sitter. The so-called "solution A," found by Einstein in 1917, represented a finite three-dimensional spatial hyperspheric surface of constant radius r embedded in 4D spacetime.

Solution A couples the initially independent parameters λ and ρ to the fixed cosmic radius r. It requires that $\lambda = c^2/r^2$ and $\rho = 2/(kr^2)$. In 1917, de Sitter invoked the second relation to arrive at $r = 8 \times 10^8$ light-years by estimating the mean cosmic mass density to be about 2×10^{-27} g/cm^3.

So it seemed for a moment that Einstein had achieved his goal of finding a finite, static universe whose size is straightforwardly determined by its mass density. But de Sitter then found another

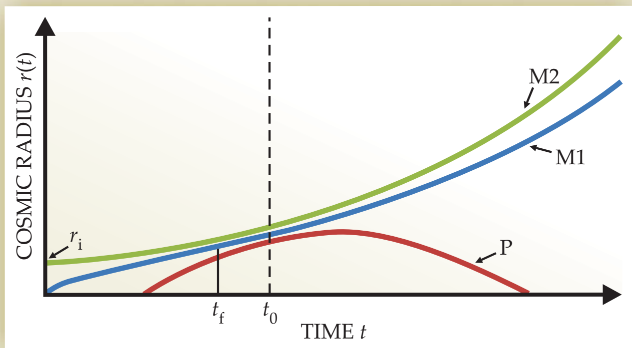


Figure 3. Three possible scenarios for cosmic evolution proposed by Alexander Friedmann[1] are shown as temporal plots of the cosmic radius. In his first monotonic scenario, M1, the cosmos expands at a decelerating rate from a zero-radius singularity until an inflection point at t_f, after which the expansion accelerates. That curve, with t_0 indicating the present, looks much like what observations have been revealing in recent decades. The second monotonic scenario, M2, shows ever-accelerating expansion from a nonzero initial radius. The periodic scenario P shows evolution from and back to zero radius.

solution that hit Einstein like a cold shower.[6] De Sitter's "solution B" presented a different sort of static universe with zero mass density and negative spatial curvature. In Einstein's solution A, all points in space are equivalent. But de Sitter's space has a unique center. Only light rays passing through that center travel along geodesics.

Einstein found de Sitter's solution unacceptable because it violated philosopher Ernst Mach's dictum that inertia cannot exist without matter. But the solution had apparent virtues. It seemed to surround every observer with a kind of horizon that might explain the redshifts in the spectra of distant galaxies that astronomers had been reporting since 1912. Furthermore, de Sitter, Eddington, and his student Lemaître looked to solution B as a way of testing GR.

Friedmann's universe

Friedmann's 1922 paper[1] cited the original papers by Einstein and de Sitter, as well as Eddington's 1920 book, *Space, Time and Gravitation*, available to him in a French edition. Instead of taking sides between Einstein and de Sitter, Friedmann approached the problem of a cosmological solution from a wider viewpoint.

His interpretation of GR shows strong grounding in Riemannian geometry. Friedmann continually reminded his readers of the need to distinguish between intrinsic features of spacetime, such as the metric, and purely mathematical artifacts like the choice of a particular coordinate representation.

The physical requirement of spatial homogeneity, he asserted, did not necessitate a static universe. Focusing on the most general form of the GR metric for a homogeneous and isotropic cosmos, Friedmann found, in addition to the static solutions A and B, a new class of nonstatic solutions of the GR field equations.[1] Like Einstein's solution A, Friedmann's solutions feature space as a 3D hypersphere. But its curvature changes in time with the hypersphere's radius $r(t)$. Now the field equations lead to a set of two ordinary differential equations for $r(t)$.

The first-order Friedmann differential equation

$$(r/c^2)(dr/dt)^2 = A - r + \lambda r^3/3c^2 \qquad (2)$$

governs the dynamics of the universe. (Nowadays $r(t)$ is regarded as an arbitrary scale length in a presumably infinite universe.) Friedmann found that the integration constant A equals $k\rho r^3/3$. Thus it's proportional to the constant total mass of the cosmos.

The rest of his 1922 paper is dedicated to analyzing the evolutionary implications of equation 2, which after integration over the cosmic radius becomes

$$t = \frac{1}{c}\int_{r_0}^{r}\sqrt{\frac{x}{A - x + \frac{\lambda}{3c^2}x^3}}\,dx + t_0. \qquad (3)$$

If one takes r_0 to be the present value, then t_0 designates, in Friedmann's words, "the time that has passed since Creation."

Three cosmic scenarios

The right side of equation 3 has physical meaning only when the cubic denominator

$$C(x) = A - x + \lambda x^3/3c^2 \qquad (4)$$

under the square-root sign is positive (see figure 2). That requirement defines three different scenarios for cosmic evolution:

▶ One gets the first scenario if $C(x)$ has no positive roots and thus is positive for all positive x. That happens when $\lambda > 4c^2/9A^2$, that is, when the cosmological constant exceeds some critical value that depends on ρ. In that case, the cosmos starts at $t = 0$ from the singularity $r = 0$, and its expansion rate changes from deceleration to acceleration at an inflection point t_f, at which $r_f = (3c^2A/2\lambda)^{1/3}$. After that, r grows asymptotically like $e^{t\sqrt{\lambda/3}}$. Friedmann called this scenario "the monotonic world of the first kind" (see the curve labeled M1 in figure 3).

▶ The second situation occurs when $0 < \lambda < 4c^2/9A^2$. In that case, $C(x)$ has two positive roots, $x_1 < x_2$, and is negative between them. This condition admits two different scenarios, 2a and 2b. In 2a, expansion oscillates between $r = 0$ and $r = x_1$. That gives the periodic solution discussed below. In the 2b scenario, expansion starts from a nonzero radius, $r_i = x_2$, and expands forever with accelerating rate. Friedmann called it the monotonic world of the second kind (curve M2 in figure 3).

▶ Friedmann called the third scenario "the periodic world" (curve P in figure 3). It results either from 2a above or from $\lambda \le 0$. In either case, $C(x)$ has only one positive root, x_1, and its interval of positivity is from 0 to x_1. The cosmos starts from the singularity $r = 0$, expands at a decelerating rate to maximum radius x_1, and then begins contracting back down to zero. The life of the cosmos is finite, ending in a Big Crunch. Assuming a total mass of 5×10^{21}

solar masses, Friedmann found a lifetime (roughly $\pi A/c$) of 10^{10} years for his periodic world.

In addition to the three principal scenarios, Friedmann also considers two special limiting cases, when λ has precisely the critical value $\lambda_c = 4c^2/9A^2$. Then $C(x)$ is degenerate; it has a double positive root at $x = 3A/2$. In one limiting case, the periodic world's expansion period becomes infinitely long, asymptotically approaching, from below, the static radius of Einstein's solution A. In the other, Friedmann's M2 world at λ_c requires an infinitely long past to rise asymptotically from Einstein's static radius, which is its r_i. Whereas these limiting cases, like Einstein's solution, bind λ to ρ, in the general Friedmann scenarios they are independent free parameters.

Einstein's reaction

When Friedmann's 1922 paper first appeared, its main ideas were mostly ignored or rejected. Einstein's immediate reaction illustrates how unwelcome the idea of a nonstatic universe was. In his view, a proper theory had to uphold the evidently static character of the cosmos.

Therefore Einstein initially found Friedmann's solution "suspicious." In September 1922, he published a short note in the *Zeitschrift für Physik* suggesting that Friedmann's derivation contained a mathematical error. In fact, Einstein had mistakenly concluded that Friedmann's equations, in the approximation that neglects pressure, imply constancy of density and therefore a cosmos of fixed size.

Learning of Einstein's note, Friedmann wrote him a long letter elaborating his derivations. But Einstein was on a world tour, returning to Berlin only in May 1923. Only then could he have read Friedmann's letter.[4] Later that month, Friedmann's Russian colleague Yuri Krutkov met Einstein at Ehrenfest's home in Leiden and clarified the confusion. So Einstein promptly published another short note in the *Zeitschrift*, acknowledging the *mathematical* correctness of Friedmann's results. He opined, however, that "the solution has no physical meaning." But wisely, he crossed out that imprudent remark from the galley proofs at the last moment. Still, it would be another eight years before Einstein was ready to accept the idea of the expanding universe.

Friedmann was the first to realize that GR alone cannot determine the geometry, topology, or kinematics of the real cosmos. The choice of one cosmological solution over another has to come from observation. For example, the universe in the shape of a finite 3D hyperspheric surface (denoted S^3 by topologists) admits "ghosts," double images of the same object in opposite directions on the sky. In 1917, de Sitter had pioneered the idea that the space of directions must be considered as the basic space, with the opposite directions viewed as one. Friedmann mentioned that view favorably,[2] and Lemaître later applied it to compute the volume of his own model cosmos.[7]

Infinite worlds

Friedmann's major concern, however, lay with the very notion of a finite cosmos, which was at the time firmly entrenched. He insisted that local metric properties alone do not resolve the problem. Inspired by Poincaré's theory of Riemannian manifolds, he imagined the possibility of a spherical universe of infinite size. The S^3 geometry did not seem to admit an infinite volume. Unabashed, however, Friedmann suggested that the azimuthal coordinate ϕ of the spherical cosmos might run not from 0 to 2π, but rather wind around over and over to infinity.

Friedmann found another, even more surprising argument to undermine the notion of a necessarily finite cosmos. On advice from his colleague Yakov Tamarkin, he sought to discover whether GR allowed solutions for a hyperboloid of infinite volume whose negative spatial curvature is given everywhere by $-6/r^2$. In such a space, every point would in effect be a saddle point. And indeed, Friedmann's 1924 paper gives a positive answer, with both static and nonstatic solutions.[2]

The static solution, like de Sitter's solution B, necessitates zero density. The nonstatic scenario, with evolving negative curvature and $r(t)$, has a nonvanishing average density whose evolution is indistinguishable from that of Friedmann's positive-curvature solution. So one can't determine the sign of the cosmic curvature simply by measuring ρ. The only change in the fundamental Friedmann equation for the negative-curvature case is that the sign of the linear term in equations 2–4 becomes positive. Friedmann thus clarified the meaning of the linear term's coefficient in $C(x)$: It gives the sign of the cosmic curvature.

The 1924 paper was also ignored. Einstein paid

additional nebula N.G.C. [illegible] observed by Pease, who found a large receding velocity but gave no numerical estimate.

RADIAL VELOCITIES OF SPIRAL NEBULAE

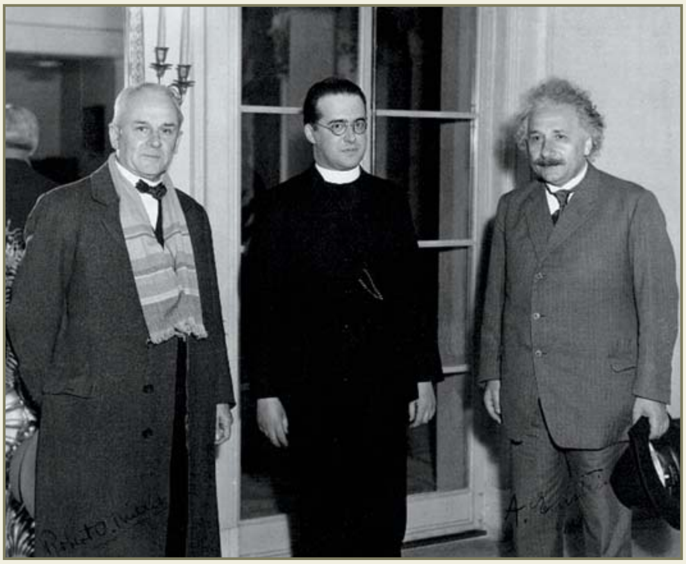
+ indicates receding, − approaching

N.G.C.	R.A. h m	Dec. ° ′	Rad. Vel. km. per sec.	N.G.C.	R.A. h m	Dec. ° ′	Rad. Vel. km. per sec
221	0 38	+40 26	− 300	4151*	12 6	+39 51	+ 980
224*	0 38	+40 50	− 300	4214	12 12	+36 46	+ 300
278†	0 47	+47 7	+ 650	4258	12 15	+47 45	+ 500
404	1 5	+35 17	− 25	4382†	12 21	+18 38	+ 500
584†	1 27	− 7 17	+1800	4449	12 24	+44 32	+ 200
598*	1 29	+30 15	− 260	4472	12 25	+ 8 27	+ 850
936	2 24	− 1 31	+1300	4486†	12 27	+12 50	+ 800
1023	2 35	+38 43	+ 300	4526	12 30	+ 8 9	+ 580
1068*	2 39	− 0 21	+1120	4565†	12 32	+26 26	+1100
2683	8 48	+33 43	+ 400	4594*	12 36	−11 11	+1100
2841†	9 16	+51 19	+ 600	4649	12 40	+12 0	+1090
3031	9 49	+69 27	− 30	4736	12 47	+41 33	+ 290
3034	9 49	+70 5	+ 290	4826	12 53	+22 7	+ 150
3115†	10 1	− 7 20	+ 600	5005	13 7	+37 29	+ 900
3368	10 42	+12 14	+ 940	5055	13 12	+42 37	+ 450
3379*	10 43	+13 0	+ 780	5194	13 26	+47 36	+ 270
3489†	10 56	+14 20	+ 600	5195†	13 27	+47 41	+ 240
3521	11 2	+ 0 24	+ 730	5236†	13 32	−29 27	+ 500
3623	11 15	+13 32	+ 800	5866	15 4	+56 4	+ 650
3627	11 16	+13 26	+ 650	7331	22 33	+33 23	+ 500
4111†	12 3	+43 31	+ 800				

The great preponderance of positive (receding) velocities is very striking; but the lack of observations of southern nebulae is unfortunate, and forbids a final conclusion. [illegible]

Figure 4. A table of line-of-sight ("radial") velocity components deduced by redshift measurements for 41 spiral galaxies compiled by Vesto Slipher and included in Arthur Eddington's 1923 book on relativity theory.[15] Each galaxy is identified by NGC catalog number and celestial coordinates. In 1927, Georges Lemaître plotted these velocities against Edwin Hubble's approximations of galaxy distances to produce the first estimate of the Hubble constant.[7]

Figure 5. At Caltech in 1933 are Robert Millikan (chair of Caltech's executive council), Georges Lemaître, and Albert Einstein. Lemaître, an ordained priest, was a physics professor at the Catholic University of Louvain, in Belgium.



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it no attention. On meeting Lemaître in 1927, he called the idea of an expanding universe "abominable." But his mind was gradually changed by growing evidence, most notably Edwin Hubble's observations of distant galaxies in 1929 and Eddington's 1930 proof that Einstein's static solution A is unstable, even with a cosmological constant.

In 1931 Einstein recognized Friedmann's achievement and suggested that his old nemesis, the cosmological constant, be expunged from GR. Einstein and de Sitter soon wrote a paper[8] promoting a flat cosmos that is just a limiting case of the Friedmann scenarios. Modern observations have as yet found no evidence of departure from Euclidean flatness on cosmological scales. And indeed such flatness is preferred by today's widely accepted inflationary Big Bang scenario. But finer observations might eventually reveal either the positive or negative curvature Friedmann put forward.

Friedmann's 1922 and 1924 papers, sadly often misquoted, have become part of the established historical narrative of the Big Bang and the accelerating universe. In his 1923 book, he suggests measuring cosmic curvature by triangulation of distant objects like Andromeda. But the book remains largely unknown, despite a recent German translation.[9]

Lemaître and Hubble

The years between Friedmann's seminal papers and the crowning revelation of accelerating cosmic expansion in 1998 witnessed two other groundbreaking achievements essential to the story: the discoveries of the Hubble constant and dark matter. Hubble's 1926 estimates of distances to distant galaxies[10] led Lemaître to formulate the Hubble constant H the following year.[7] In 1931, Lemaître first gave Friedmann's singularity a physical meaning,[11] that of a "primeval atom" blowing up—what Fred Hoyle later dismissively called "the Big Bang."

Unable to foresee the 1998 discovery of cosmic acceleration, Einstein no longer saw any need for a cosmological constant. He thought Friedmann's expanding solution with $\lambda = 0$ might be the answer. Starting with the 1946 edition of his popular exposition *The Meaning of Relativity*, Einstein interjected that "the mathematician Friedmann found a way out of this [cosmological constant] dilemma. His result then found a surprising confirmation in Hubble's discovery of the expansion of the stellar system. . . . The following [15-page exposition] is essentially nothing but an exposition of Friedmann's idea."[12] Unfortunately, Einstein attributed to Hubble alone what properly belongs to several people, among them de Sitter and Vesto Slipher.

Later generations have bestowed the title "father of modern cosmology" primarily on Lemaître or Hubble.[13] The debate among historians has focused on the portion of Lemaître's 1927 paper[7] that contains his introduction of the Hubble constant. Strangely, that portion was omitted in the 1931 English translation.[14] Still, the consensus is that the "Hubble" constant was solely Lemaître's idea.

Although Lemaître was unaware of Friedmann's 1922 and 1924 papers, he appeared on the scene just when the shortcomings of the static solutions A and B were becoming clear in the light of the data coming from the new 100-inch telescope on Mount Wilson near Los Angeles. Unlike Friedmann, Lemaître was in possession of Hubble's 1926 galaxy-distance data and Eddington's 1923 book on GR and cosmology.[15] That book brought Lemaître's attention to the spectral redshifts of 41 spiral galaxies measured by Slipher (see figure 4).

Plotting the line-of-sight velocity component deduced from each galaxy's redshift against Hubble's estimate of its distance d, Lemaître postulated that they were proportional to each other, and he found a best fit for the proportionality constant H. His major contribution was to connect H to the evolving nonstatic cosmic radius via $rH = dr/dt$. Thus, measuring the Hubble constant yields an estimate of the age of the universe.[7] Certainly a great achievement, but not, I would argue, meriting the paternity of Big Bang cosmology.

In fact, Lemaître missed the Big Bang solution in his 1927 paper. Having rediscovered the Friedmann equations, he failed to consider all classes of solutions. Instead, he considered only the limiting case discussed above, in which $C(x)$ has a double positive root. He identified that root with r_i, a finite initial cosmic radius like that of Friedmann's M2 scenario. But unlike the M2 scenario, Lemaître's solution required a critical value of λ specified by the total mass of the cosmos.

Lemaître stuck with the finite-initial-radius limiting case for years after Einstein (shown with Lemaître in figure 5) introduced him to Friedmann's papers in 1927. Only in 1931 did he begin to consider the Big Bang scenario.[11] So it's puzzling that historians Harry Nussbaumer and Lydia Bieri recently concluded that "Lemaître owes nothing to Friedmann."[5] Indeed, "nothing" except for the idea that the cosmological constant is a fully independent parameter and that the universe was born in a singularity.

Ironically, the idea of an initial singularity was subverted for decades by early attempts to measure H. Greatly underestimating the distances to remote galaxies, Hubble understated the age of the universe by an order of magnitude. Einstein, in his last years, despaired of finding a way out of the paradox of a cosmological age of less than 2 billion years and a geological age that exceeded 4 billion! Only after Einstein's death in 1955 did the magnitude of Hubble's error become clear.

Confirmation and legacy

The early favorite among Friedmann's three principal scenarios was the periodic world (P in figure 3). It allowed multiple cosmic births and deaths—reminiscent of Greek and Asian philosophies of reincarnation. But by the early 1990s, cosmologists generally assumed that the cosmos was flat, and that its expansion rate was asymptotically slowing to zero, with no cosmological constant to resist the pull of gravity. So the 1998 results that revealed an accelerating expansion came as a great surprise, deemed worthy of the 2011 Nobel Prize.

Teams led by the three laureates—Saul Perlmutter, Adam Riess, and Brian Schmidt—discovered the acceleration by exploiting distant type 1a supernovae as standard candles. But the 1998 results, by themselves, could not discriminate between Friedmann's M1 and M2 monotonic worlds.

The litmus test invokes Friedmann's formula for the cosmic radius r_f at the inflection point in M1. In modern notation, it reads

$$r_f = r_0\,(\Omega_M/2\Omega_\Lambda)^{1/3}, \tag{5}$$

where Ω_M and Ω_Λ are the present mean cosmic energy densities due, respectively, to matter and λ, both normalized to the critical total energy density required by inflationary cosmology. With the approximate values—$\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $t_0 = 14$ billion years—determined by a reassuring convergence of cosmological data, equation 5 says that the inflection from deceleration to acceleration should have happened about 5.6 billion years ago. Only in 2004 was the issue resolved, when Riess and coworkers confirmed the M1 prediction by measuring ultrahigh-redshift supernovae from the epoch of cosmic deceleration (see PHYSICS TODAY, June 2004, page 19).

There has been a tendency to present Friedmann as merely a mathematician, unconcerned with the physical implications of his discovery.[13] But such a view is belied even by Friedmann's considerable achievements in meteorology and aerodynamics. The wide spectrum of the problems he solved, as seen in his collected works,[16] leaves no doubt that he cared about verification of his theories. His death at age 37 prevented him from seeing any of the observational triumphs of his pioneering cosmological ideas. I contend that his early death has contributed to the undervaluation and misrepresentation of his contributions to modern cosmology.

It's clear that in adumbrating Big Bang cosmology, Friedmann went much further than his predecessors or early successors like Lemaître. He liked to quote Dante's line: "*L'acqua ch'io prendo già mai non si corse*" (The sea I am entering has never yet been crossed). His approach, the first correct application of GR to cosmology, introduced the idea of an expanding universe, possibly born from a singularity. Moreover, realizing that GR admits a variety of cosmological metrics, Friedmann first alerted physicists to the possibility that the cosmos might be negatively curved and infinite in size.

Still, after the 1930s, Lemaître received almost all the credit for the Big Bang theory. But the voices of Russian physicists speaking out on behalf of Friedmann's achievements were ultimately heard. One of them, Yakov Zeldovich, wrote that

> Friedmann published his works in 1922–1924, a time of great hardships. In the issue of the 1922 journal that carried Friedmann's paper, there was an appeal to German scientists to donate scientific literature to their Soviet colleagues, who were separated from it during the revolution and the war. Friedmann's discovery under those conditions was not only a scientific but also a human feat![16]

I thank Larry Horwitz, Alexei Kojevnikov, Zinovy Reichstein, Robert Schmidt, Reinhold Bien, Dierck Liebscher, Leos Ondra, Evgeny Shapiro, and Eduardo Vila Echagüe for helpful discussions.

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